

Mimicking general relativity in the solar system

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In order for a modified gravity model to be a candidate for cosmological dark energy it has to pass stringent local gravity experiments. We find that a Brans-Dicke (BD) theory with well-defined second order corrections that include the Gauss-Bonnet term possess this feature. We construct the generic second order theory that gives, to linear order, a BD metric solution for a point-like mass source. We find that the Eddington parameter γ , heavily constrained by time delay experiments, can be arbitrarily close to the GR value of 1, with an arbitrary BD parameter ω_{BD} . We find the region where the solution is stable to small timelike perturbations.

Brans-Dicke (BD) theory is a simple modification of general relativity (author?) [1] (see also (author?) [2] for generalisations) as it is a single massless scalar-tensor theory whose only parameter is the kinetic coupling term ω_{BD} ,

$$S_{\text{BD}} = \int \sqrt{-g} \left[\Phi R - \frac{\omega_{\text{BD}}}{\Phi} (\nabla \Phi)^2 \right] - 16\pi \mathcal{L}_{\text{matter}}. \quad (1)$$

Its GR limit is obtained for $\omega_{\text{BD}} \rightarrow \infty$. BD gravity breaks the strong equivalence principle and yields at local scales differing Eddington parameters β and γ to those of GR, which are strictly equal to 1. In particular the parameter γ , which measures how much spatial curvature is produced by unit rest mass (see (author?) [3]), is given by $\gamma = (1 + \omega_{\text{BD}})/(2 + \omega_{\text{BD}})$. It is strongly constrained by time delay experiments, such as the one conducted with the Cassini spacecraft, which recently gave $|\gamma - 1| \lesssim 10^{-5}$ (author?) [4] (for a recent review and alternative methods to measure γ see (author?) [3]). This implies $\omega_{\text{BD}} > 40000$, therefore the scalar sector is very weakly coupled.

On the other hand modification of general relativity is possible or even needed in order to explain effects on cosmological and galactic scales, or at scales just beyond the solar system (such as the Pioneer anomaly (author?) [5]). At cosmological scales, some 10^{15} times bigger than solar system scales, supernovae data (author?) [7] entertain the possibility that GR may be modified at large distances (author?) [6]. Modification of GR is also being envisaged at galactic scales in order to explain deviations from standard Newtonian gravity in galactic rotation curves, as in MOND or Bekenstein-Sanders theor (author?) [8]. Scalars have been quite naturally introduced in order to mediate gravity modification or even as sources of cosmological dark energy (author?) [9]. These modifications are well into the classical, infrared sector of gravity at very low energies, very far from the the Planck scale UV sector, where quantum gravity becomes important. Even from the point of view of UV modifications, string theory predicts a zoo of scalars that, if massless, would give BD-type phenomenology in the so-

lar system. It is therefore quite fair to say that there is increasing tension between gravitational constraints imposed experimentally for weak gravity in the solar system and laboratory tests, as well as strong gravity from binary pulsars (see for example (author?) [10]), and on the other hand theories of modified gravity or dark energy that aim to explain unexpected outcomes of novel experimental data. The aim of this letter is to show that well motivated second order corrections to simple BD theory (with no potential) can mimic GR at the solar system scale, in the sense of giving $\gamma = 1$ independently of the parameter ω_{BD} . This does not mean that the toy scalar-tensor theory in question and GR would not be distinguishable, quite the contrary, on cosmological scales their phenomenology would be totally different and even at the solar system level one could detect some effect, most probably by carrying out an experiment to measure β , as we will discuss in the concluding remarks.

Our starting point is the general (modulo field redefinitions) scalar-tensor Lagrangian of second order in powers of the curvature tensor which has the unique property of giving second order field equations,

$$\mathcal{L} = \sqrt{-g} [f_1 R - f_2 (\nabla \phi)^2 + \xi_1 \mathcal{L}_{GB} + \xi_2 G^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi + \xi_3 (\nabla \phi)^2 \nabla^2 \phi + \xi_4 (\nabla \phi)^4 - 2V] - 16\pi G_0 \mathcal{L}_{\text{matter}}. \quad (2)$$

The theory (2) is parametrised by the potential V and couplings f_i, ξ_i which are all functions of the scalar field ϕ . The Gauss-Bonnet term is $\mathcal{L}_{GB} = R_{\mu\nu\sigma\rho} R^{\mu\nu\sigma\rho} - 4R_{\mu\nu} R^{\mu\nu} + R^2$ and is a topological invariant in 4 dimensions for GR but not for scalar-tensor (2). Theories of the above form (2) have been proposed as a solution to the dark energy problem (author?) [11]. For the case of $f_1 = 1$, it has been shown that solar system data can impose severe restrictions on the couplings ξ_i (author?) [12], which allows the range of possible gravity modification to be narrowed down. Throughout this article we work in units with $c = 1$. We will assume $\xi_3 = \xi_4 = 0$ in the following as a compromise between introducing new free coupling functions in the theory and generality of the setup. We choose to keep the non-minimal interaction terms between the graviton and the scalar ξ_1 and ξ_2

rather than higher order scalar corrections. When searching for solutions we will assume also a vanishing potential V (as in BD) since invoking a large mass is an obvious but totally *ad hoc* way to evade solar system constraints and would suppress observable effects at all scales. Our work also contrasts with ‘chameleon’ models (author?) [13], in which the scalar’s gravity is suppressed at local (but not cosmological) scales by a suitably chosen potential that yields a background dependent mass in eq. (2). The higher order corrections are chosen so that the gravitational propagator does not pick up extra degrees of freedom. Therefore the resulting field equations will be a-priori ghost-free around the Minkowski vacuum, second order in the derivatives, with well-defined Dirac distributional terms.

In order to derive the post-Newtonian equations, we now assume a static, spherically symmetric metric in isotropic coordinates,

$$ds^2 = -(1 - 2U)dt^2 + (1 + 2\Upsilon)\delta_{ij}dx^i dx^j + \mathcal{O}(\epsilon^{3/2}), \quad (3)$$

where U is the Newtonian potential and Υ is the leading post-Newtonian spatial contribution. They are both functions of the radial co-ordinate r only and are assumed to be of the order of the smallness dimensionless parameter $\epsilon = Gm_\odot/r$ where m_\odot is the solar mass and r is a characteristic length scale of the problem. For the solar system $\epsilon \lesssim 10^{-5}$ for r greater than the sun’s radius. The post-Newtonian parameter (PPN) we will be calculating is Eddington’s parameter defined as $\gamma = \Upsilon/U$. Matter energy density ρ_m is given by the mass of the sun and is as usual assumed to be a distributional source at $r = 0$: $\rho_m = m_\odot \delta^{(3)}(x)$. This is an excellent assumption given that the Schwarzschild radius of the sun is of the order of 3 km compared to scales of the order of astronomical units. Note however, that higher order terms in ϵ have to be included in (3) in order to calculate β (for the advance of Mercury’s perihelion for example). For the relativistic experiment we will consider here, namely time delay, our expansion is necessary and sufficient. Let us now define the operators,

$$\Delta F = \sum_i F_{,ii}, \quad \mathcal{D}(X, Y) = \sum_{i,j} X_{,ij} Y_{,ij} - \Delta X \Delta Y, \quad (4)$$

which will be the technical tool essential for our analysis. For functions with only r -dependence they reduce to $\Delta F = r^{-2} \partial_r (r^2 \partial_r F)$ and $\mathcal{D}(X, Y) = -2r^{-2} \partial_r (r \partial_r X \partial_r Y)$ and in particular,

$$\begin{aligned} \mathcal{D}(r^{-n}, r^{-m}) &= \frac{2nm}{n+m+2} \Delta r^{-(n+m+2)}, \\ \Delta r^{-n} &= \frac{n(n-1)}{r^{n+2}} - \frac{4\pi n \delta^{(3)}(x)}{r^{n-1}}, \end{aligned} \quad (5)$$

and thus we can easily evaluate the relevant distributional parts associated with \mathcal{D} . We do not make any

assumptions about the relative sizes of f_i , ξ_i , V , or their derivatives (since we expect the higher order terms to play a significant role), and instead include the leading order in ϵ contribution from each term in the field equations

$$\begin{aligned} f_1 \Delta U &= -4\pi G_0 \rho_m + V + \frac{\Delta f_1}{2} - 2\mathcal{D}(U + \Upsilon, \xi_1) \\ &\quad + \mathcal{O}(\epsilon^2, f_2, \epsilon V, \epsilon \xi_2, \epsilon^2 \xi_1) \end{aligned} \quad (6)$$

$$\begin{aligned} f_1 \Delta \Upsilon &= -4\pi G_0 \rho_m - \frac{V}{2} - \frac{\Delta f_1}{2} - 2\mathcal{D}(\Upsilon, \xi_1) + \frac{\mathcal{D}(\phi, \zeta_2)}{4} \\ &\quad + \mathcal{O}(\epsilon^2, \epsilon V, f_2 \epsilon \xi_2, \epsilon^2 \xi_1) \end{aligned} \quad (7)$$

where we have defined $\xi_2 = \partial_\phi \zeta_2$ and $f_2 = \partial_\phi h_2$. We will also expand f_1 to first order in ϵ , $f_1 = \Phi_0 + \mathcal{O}(\epsilon)$. The scalar field equation on the other hand is globally of one order higher and gives to leading order

$$\begin{aligned} \partial_\phi h_2 \Delta \phi + \Delta h_2 &= 2\partial_\phi V + 2\partial_\phi f_1 \Delta(2\Upsilon - U) \\ &\quad - 8\partial_\phi \xi_1 \mathcal{D}(U, \Upsilon) + \mathcal{D}(U - \Upsilon, \zeta_2) \\ &\quad + \partial_\phi \zeta_2 \mathcal{D}(U - \Upsilon, \phi) + \mathcal{O}(\epsilon^2, \epsilon V, f_2 \epsilon, \epsilon^3 \xi_1, \epsilon^2 \xi_2). \end{aligned} \quad (8)$$

For comparison, BD theory ($V, \xi_i \equiv 0$) has

$$f_1 = \Phi \equiv \Phi_0 + \phi, \quad f_2 \equiv \frac{\omega_{\text{BD}}(\Phi)}{\Phi} \approx \frac{\omega_{\text{BD}}}{\Phi_0} + \mathcal{O}(\phi) \quad (9)$$

Since we are expanding the equations to the lowest non-trivial order, we cannot estimate the second PPN parameter β , which requires higher order terms in the metric coefficients. This loss in generality in the metric coefficients is compensated by the great generality of our solution.

It is useful to define $\eta \equiv 1 - \gamma$ whereupon the various constants in the model are related by $\omega_{\text{BD}} = -2 + 1/\eta$ and $G = 2G_0/[(2 - \eta)\Phi_0]$. We see that $\eta = 0$ gives exactly GR. We are interested in gravitational theories emanating from (2), which while not identical to general relativity, give almost identical predictions for the weak field of the solar system. One approach to this problem would be to solve the field equations of the previous section for a range of coupling functions ξ_i , f_i , and then compare the resulting potentials U and Υ , with those of Einstein gravity. We will not take this approach since we have no interest in the solutions of the field equations, except for the special cases where they give (to this order in ϵ) precisely the Newtonian result $U \approx \Upsilon \approx Gm_\odot/r$. This in particular gives us agreement with tests of Newton’s law from planetary orbits. Therefore, instead of trying to find the metric (3) which solves the field equations for given ξ_i , f_i , we will inversely start by assuming the desired Newtonian form of U and Υ , and then view (6)–(8) as equations for the coupling functions f_i , ξ_i parametrisng the theory (2). As discussed above, we will now set $V = 0$, just as in the standard BD model and we allow the PPN parameter $\gamma = 1 - \eta$ to take any value. Hence

we take

$$U = \frac{Gm_\odot}{r}, \quad \Upsilon = (1 - \eta) \frac{Gm_\odot}{r}. \quad (10)$$

Note also that the effective gravitational coupling G need not be equal to the fundamental parameter for the gravitational coupling of matter G_0 . The general solution of (6)–(8) is then

$$\begin{aligned} f_1 &= \Phi_0 + Gm_\odot \left(\frac{\Phi_0 \eta + \lambda}{r} + \frac{2 - \eta}{2} \int \frac{\partial_r S}{r} dr \right), \\ r^4 (\partial_r \phi)^2 f_2 &= (Gm_\odot)^2 [\Phi_0 \eta + \lambda - 2(\Phi_0 + \lambda) \eta^2 \\ &\quad - 3(1 - 4\eta + 2\eta^2)S - \eta(3 - 2\eta)r \partial_r S], \\ \xi_1 &= -\frac{\eta \lambda r^2}{8} + \int \frac{r^2 \partial_r S}{16} dr, \\ (\partial_r \phi)^2 \xi_2 &= Gm_\odot \left[2 \frac{\lambda}{r} (1 - \eta) - \frac{3 - 2\eta}{2} \partial_r S \right], \end{aligned} \quad (11)$$

where λ is an arbitrary, dimensionful constant, obtained by the distributional part appearing in the equations of motion (6)–(8) as the boundary condition at $r = 0$. On the other hand, $S(r)$ is an arbitrary function with the regularity condition $rS' = 6S$ as $r \rightarrow 0$, i.e. $S = r \partial_r S = 0$ at $r = 0$. Viewing the above expressions as the solution of ordinary inhomogeneous differential equations for f_i , ξ_i , the function $S(r)$ parametrises the general homogeneous solution of (6)–(8) whereas λ parametrises the particular solution. The integrals range from ∞ to r . The gravitational coupling satisfies

$$\frac{G_0}{G} = \Phi_0 \left[1 - \frac{\eta}{2} \right] - \frac{\lambda}{2} [1 - 4\eta + 2\eta^2]. \quad (12)$$

The above equations fully specify the couplings needed to reproduce a PPN parameter γ and an exactly Newtonian $1/r$ gravitational potential. In fact λ and S now parametrise the theory (2). Setting $S = \lambda = 0$ gives us pure BD (9) with $\phi = Gm_\odot \Phi_0 \eta / r$. Setting on top of that $\eta = 0$ gives GR. The key point however is that if we set $\eta = 0$ keeping S and λ non-zero we have *the same post-Newtonian limit as standard GR*, i.e. $\gamma = 1$ in (3) without the theory actually being GR. Indeed note that the corresponding kinetic coupling f_2 can take arbitrary values parametrised by λ and S . Indeed when the higher curvature terms are included, the Newtonian potential is still proportional to $1/r$. For non-trivial S , this is because all the corrections to standard gravity cancel out, making the gravity modifications ‘invisible’ to this order. On the other hand, if λ is non-zero, the corrections do not cancel, but instead ‘mimic’ Newtonian gravity. This can be seen from the fact that the effective gravitational coupling G receives a λ dependent correction (12). A similar effect was found for $f(\mathcal{L}_{GB})$ gravity in (author?) [14], although the resulting γ was too large.

This fact is made clearer when we note that the above solution (11) does not give a specific form for ϕ . This is natural, since by a change of variables, ϕ can be made

to take any desired form. Since we wish to express the functions in terms of ϕ , let us take

$$\phi = \phi_1 \frac{r_g}{r}. \quad (13)$$

The constant ϕ_1 simply corresponds to a re-scaling of ϕ . Defining $r_g = Gm_\odot$, the expressions (11) then give

$$\begin{aligned} f_1 &= \Phi_0 + (\Phi_0 \eta + \lambda) \frac{\phi}{\phi_1} + \frac{2 - \eta}{2\phi_1} \int \phi \partial_\phi S d\phi, \\ f_2 &= \frac{1}{\phi_1^2} [\Phi_0 \eta + \lambda - 2(\Phi_0 + \lambda) \eta^2 \\ &\quad - 3(1 - 4\eta + 2\eta^2)S + \eta(3 - 2\eta)\phi \partial_\phi S], \\ \xi_1 &= \frac{r_g^2 \phi_1^2}{8} \left[-\frac{\eta \lambda}{\phi^2} + \int \frac{\partial_\phi S}{2\phi^2} d\phi \right], \\ \xi_2 &= r_g^2 \phi_1 \left[\frac{2\lambda}{\phi^3} (1 - \eta) + \frac{3 - 2\eta}{2\phi^2} \partial_\phi S \right], \end{aligned} \quad (14)$$

with the regularity conditions at $r = 0$ now implying S and $\phi \partial_\phi S$ tending to zero as $\phi \rightarrow \infty$. S can then be expanded as $S = \sum_{n \geq 1} c_n \phi^{-n}$, obtaining the general asymptotic solution to all orders in ϕ . Note also that the higher order couplings are r_g dependent which follows from the fact that they are of dimension length squared. In particular this means that if we introduce the Gauss-Bonnet coupling constant α then it is related via a multiplicative number to the only length scale of the problem r_g , namely, $\alpha = -\phi_1^2 \eta \lambda r_g^2 / 8$. In fact the multiplicative constant is the hierarchy generated between the classical scale r_g and $\sqrt{\alpha}$.

To illustrate our result we consider the simplest case of $S = 0$, and take $|\eta| < 10^{-5}$ to agree with solar system constraints. Without loss of generality we set $\phi_1 = \lambda + \Phi_0 \eta$. We see that even with $\eta = 0$ we have a BD Lagrangian (2) with the additional term $\xi_2 G^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi$ which can reproduce general relativity up to the first post-Newtonian parameter γ . Therefore finding $\gamma = 1$ does not guarantee the absence of a scalar interaction even in this simplest of cases since the scalar coupling ω_{BD} is still freely given by Φ_0 / λ . Note also that the strength of the scalar interaction ω_{BD} is inversely proportional to the strength of the higher order corrections, as parametrised by λ .

We will now examine the stability of the solution (14) with respect to time-dependent perturbations. We take $U \rightarrow U + \delta U$, etc. and keep the leading order time derivatives of δU (up to $\mathcal{O}(\delta U)$, for linear gravity terms, and $\mathcal{O}(\epsilon \delta U)$ for quadratic terms). For simplicity we will restrict ourselves to the extreme case of solution (14) with $\eta = 0$ and $S \equiv 0$. The corresponding BD-like parameter for the higher order theory is $\omega_{BD} = \Phi_0 / \lambda$. The perturbation equations then reduce to

$$3\omega_{BD} \delta \ddot{\Upsilon} - \frac{3}{2} \frac{\delta \ddot{\phi}}{\phi_1} = -\omega_{BD} \Delta \delta U + \frac{1}{2} \Delta \frac{\delta \phi}{\phi_1} \quad (15)$$

$$\begin{aligned}
-\omega_{\text{BD}}\Delta\delta\Upsilon + \frac{1}{2}\Delta\frac{\delta\phi}{\phi_1} &= -\frac{1}{r^2}\partial_r\left(\frac{\partial_r(r^3\delta\phi)}{r\phi_1}\right) \\
-3\delta\ddot{\Upsilon} - \frac{\delta\phi}{\phi_1} &= \Delta\delta U - \Delta\frac{\delta\phi}{\phi_1} + 2r\partial_r\left(\frac{\partial_r(\delta U - \delta\Upsilon)}{r}\right)
\end{aligned}
\tag{16}$$

(17)

where we have assumed that the perturbations are more regular than the leading order solution $1/r$ as $r \rightarrow 0$. If (14) is to be a viable gravity model, there needs to be a reasonable range of parameters for which δU , etc. oscillate, rather than growing over time. Substituting $\delta U(t, \vec{r}) = \delta U(t) \exp i \vec{k} \cdot \vec{r}$ and analogously for $\delta\phi, \delta\Upsilon$, we find

$$\ddot{\delta\phi} = -\frac{2\omega_{\text{BD}} + 3}{2\omega_{\text{BD}} + 9 - \frac{18k^3 r^3 (kr-3)}{(k^2 r^2 + 4kr-4)^2}} k^2 \delta\phi \rightarrow \frac{2\omega_{\text{BD}} + 3}{9 - 2\omega_{\text{BD}}} k^2 \delta\phi
\tag{18}$$

(the limit is for $r \rightarrow \infty$). In the same limit, the time-dependence of $\delta U, \delta\Upsilon$ is the same as for $\delta\phi$.

If $\omega_{\text{BD}} > 9/2$ or $\omega_{\text{BD}} < -3/2$, the perturbations will oscillate for large r , rather than grow exponentially, indicating that our gravitational solution mimicking GR is classically stable for a reasonable range of parameters, at least at first order. Although it might be that including higher order terms in the perturbation expansion the oscillations turn out to be of a growing nature, one has to be reminded that the oscillations will be naturally damped by the emission of gravitational waves and by the background cosmological expansion. A further constraint comes from requiring positive gravitational coupling. For the above case, (12) reduces to $G = (G_0/\Phi_0)2\omega_{\text{BD}}/(2\omega_{\text{BD}} - 1)$. Hence $G > 0$ implies $\omega_{\text{BD}} < 0$ or $\omega_{\text{BD}} > 1/2$, which are already covered by the above ranges. We expect qualitatively similar results for more general (14) with η or $S(\phi)$ non-zero, although a proof of this is beyond the scope of this paper. Let us remember that $S(r)$ is anyway an arbitrary function and one can always choose it a posteriori in such a way to maintain stability.

In this paper we exhibited a sensible second order (in powers of the curvature tensor not derivative) scalar-tensor theory which shares some characteristics of ordinary BD or GR, in particular, well-defined second order field equations, distributional boundary conditions and well defined stable vacua. We found that such a theory, given the right coupling functions, can mimic a GR Eddington parameter γ exactly equal to 1 with virtually no constraint on the kinetic coupling ω_{BD} (in ordinary BD theory actual measurements of γ give $\omega_{\text{BD}} > 40000$). In this sense we saw that the inclusion of higher order operators in the action can mimic GR with a scalar-tensor theory. We do not view the solutions we have found (14) or even the model in question (2), as some fundamental scalar-tensor theory; our aim was rather to see how robust were the solar system predictions to higher order corrections. Our conclusion is that certain solar system constraints known to rule out theories such as BD

are not as robust in their GR prediction as one might think. In fact similar results have been shown for certain vector-tensor theories (**author?**) [15] although one expects closer agreement with the Eddington parameters in the case of vectors rather than scalars.

This does not mean that one cannot distinguish between such higher order scalar tensor theories and GR. For a start the second PPN parameter β may not be unity for such theories, although if we allow for the remaining higher order operators ξ_3, ξ_4 in (2), in principle we have the mathematical flexibility in the equations to again fix $\beta = 1$ by solving for the coupling functions. The main difference between GR, BD and these higher order theories is that the coupling functions are dimensionful. Thus the relevant solutions such as (14) will depend on the length scale of the solution, namely r_g , times some dimensionless number whose magnitude will determine the “fine-tuning” one has to impose between the length scales of the theory and the solar system. In other words we would view the experimental error bars as hierarchies between the higher order couplings and local scales where the experiment is carried out. This relation may also be relaxed by allowing for the general second order theory at the expense of introducing further free parameters in the theory (2). We further note that we have constructed gravitational theories which exactly reproduce the Newtonian potential for the sun: $U = r_g/r$. In fact it is perfectly acceptable to have $U - r_g/r$ non-zero, but smaller than the experimental bounds from planetary orbits. The above issues as well as a calculation of β , other observational signatures of such higher order theories, in the laboratory or in the solar system, and their cosmology are open interesting questions which we hope shall be addressed in the near future.

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